# Design of a Confidence Interval Observer Application to Vehicle Positioning

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## INTRODUCTION

### Objectives

- **Failsafe Reconstruction of vehicle position, velocity and acceleration**
- **Algorithm Design in critical security applications (ex. Collision avoidance)**
- $p(x(t) \in [x^-(t), x^+(t)] \geq \beta_{obj}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ : $x_1$ measured state, $x_2$ unmeasured state

$u$ : inputs

$\theta$ : parameters

$y$ : outputs

### Preliminaries

- $\dot{x}(t) = f(x(t), u(t), \theta) \quad x(0) = x_0$

  $y_k = x_1(t_k)$

  $p(x_0 \in [x_0^-, x_0^+]) \geq \beta_0$

  $p(\theta \in [\theta^-, \theta^+]) \geq \beta_\theta$

  $p(u(t) \in [u_k^-, u_k^+]) \geq \beta_{u_k} \quad t_k \leq t \leq t_{k+1}$

  $p(x_1(t_k) \in [y_k^-, y_k^+]) \geq \beta_{y_k}$

- Boolean Random Variables
- $p(a \in [a_1^-, a_1^+] \cap [a_2^-, a_2^+]) \geq$
  - ind. $\beta_{a_1}\beta_{a_2}$
  - dep. $\beta_{a_1} + \beta_{a_2} - 1$
- $p(a \in [a_1^-, a_1^+] \cup [a_2^-, a_2^+]) \geq$
  - ind. $\beta_{a_1} + \beta_{a_2} - \beta_{a_1}\beta_{a_2}$
  - dep. $\max(\beta_{a_1}, \beta_{a_2})$

## CONFIDENCE INTERVAL OBSERVER STRUCTURE

### Prediction Step $(t_k \leq t \leq t_{k+1})$

- **General structure**

Two coupled dynamics systems

$\dot{x}_P^+ = f^+(x_P^+, x_P^-, u^+, u^-, \theta^+, \theta^-) \quad x_P^+(t_k) = x^+(t_k)$

$\dot{x}_P^- = f^-(x_P^+, x_P^-, u^+, u^-, \theta^+, \theta^-) \quad x_P^-(t_k) = x^-(t_k)$

Definition of the error vector

$$e(t) = \begin{bmatrix} e^+(t) \\ e^-(t) \end{bmatrix} = \begin{bmatrix} x_P^+(t) - x(t) \\ x(t) - x_P^-(t) \end{bmatrix} \geq 0$$

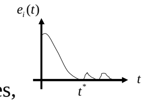
- **Positivity of the error vector**

Assuming $e(t_0) \geq 0$, considering the $i^{th}$ component of $e(t)$

If $\exists t^* \geq t_0$ such that $e_i(t^*) = 0$

$e_j(t^*) \geq 0 \quad \forall j \neq i \in \{1, ..., 2n_x\}$

then $\dot{e}_i(t^*) \geq 0$




Every time a component $e_i$ vanishes, its derivative has to be $\geq 0$ to prevent $e_i < 0$

- **Guaranteed enclosure of the predictor**

It is sufficient for all the variables and the parameters to be bounded at the present time to guarantee bounded intervals on state variables at future instants.

- **Lower bound of the integrity level**

$p(x(t) \in [x_P^-(t), x_P^+(t)]) \geq \beta_{x_k}\beta_\theta\beta_{u_k}$

$t \geq t_k \qquad \geq \beta_{obj}$

$p(x(t_k) \in [x_P^-(t_k), x_P^+(t_k)]) \geq \beta_{x_k}$

### Correction Step $(t_k)$

- **Interval combinations**
  - Using failsafe sensors and predictor
  - Every possible combinations based on union and intersection operations
  - Integrity computed using appropriate assumptions on dependence (or independence) of the intervals
  - The shortest non-empty interval satisfying the integrity objective is chosen

- **Example**

Combinations set of : $[a_1], [a_2], [a_3], [a_4]$

Independence assumption and $\beta_{a_1} = \cdots = \beta_{a_4} = 10^{-4}$

k : number of possible union operations

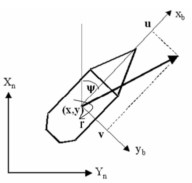
h : number of possible intersection operations

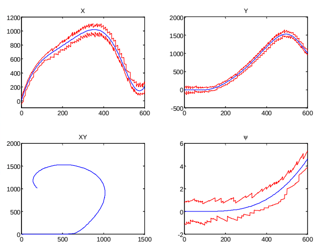
- $k=1, h=1$: $[a_1], [a_2], [a_3], [a_4]$, $\beta = 1-10^{-4}$
- $k=1, h=2$, $\beta \approx 1-2\times10^{-4}$: $[a_1]\cap[a_2], [a_1]\cap[a_3], [a_1]\cap[a_4]$; $[a_2]\cap[a_3], [a_2]\cap[a_4], [a_3]\cap[a_4]$
- $k=1, h=4$: $[a_1]\cap[a_2]\cap[a_3]\cap[a_4]$, $\beta \approx 1-4\times10^{-4}$
- $k=1, h=3$, $\beta \approx 1-3\times10^{-4}$: $[a_1]\cap[a_2]\cap[a_3], [a_1]\cap[a_2]\cap[a_4]$; $[a_1]\cap[a_3]\cap[a_4], [a_2]\cap[a_3]\cap[a_4]$
- $k=2, h=1$, $\beta = 1-10^{-8}$: $[a_1]\cup[a_2], [a_1]\cup[a_3], [a_1]\cup[a_4]$; $[a_2]\cup[a_3], [a_2]\cup[a_4], [a_3]\cup[a_4]$
- $k=2, h=2$, $\beta \approx 1-2\times10^{-8}$: $([a_1]\cup[a_2])\cap([a_3]\cup[a_4])$; $([a_1]\cup[a_3])\cap([a_2]\cup[a_4])$; $([a_1]\cup[a_4])\cap([a_2]\cup[a_3])$
- $k=3, h=1$, $\beta = 1-10^{-12}$: $[a_1]\cup[a_2]\cup[a_3], [a_1]\cup[a_2]\cup[a_4]$; $[a_1]\cup[a_3]\cup[a_4], [a_2]\cup[a_3]\cup[a_4]$
- $k=4, h=1$: $[a_1]\cup[a_2]\cup[a_3]\cup[a_4]$, $\beta = 1-10^{-16}$

## SIMULATION EXAMPLE AND CONCLUSIONS

### Simulation Example

**Autonomous Underwater Vehicle**




$\eta = [x, y, \psi]^T, \; \nu = [u, v, r]^T$

$\dot{\eta} = J(\psi)\nu$

$M\dot{\nu} + C(\nu)\nu + D\nu = \tau$

$$J(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### Conclusions

- **The computed intervals enclose state variables**
- **The integrity level can be reached to satisfy security demand in critical applications**
- **Need to develop fault detection algorithm to detect aberrant measurements**

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