Classical Models of Secondary Settlers Revisited

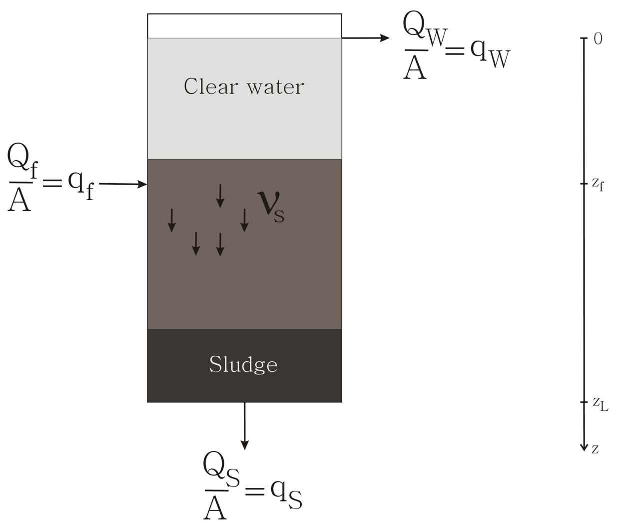
R. David and A. Vande Wouwer
Service d'Automatique, FPMs, Belgium, Email:*Alain.VandeWouwer@fpms.ac.be*

J.- L. Vasel
Service Assainissement et Environnement, ULg, Site d'Arlon, Belgium, Email:*JLVasel@ulg.ac.be*

Objective and approach of this study

- **Review of existing models based on a "mixed-tank-in-series" formulation**
- **Model analysis and improvement, particularly:**
  - ✓ **definition of the sedimentation velocity**
  - ✓ **spatial discretization and formulation of the boundary conditions**
- **Numerical simulation study following a method of lines approach (using the Matlab library MATMOL)**

Secondary settler modelling



The performance of the activated sludge process (in a wastewater treatment plant) strongly depends on the performance of the secondary settler, which separates the treated effluent (Q_W) from the activated sludge (Q_S) thanks to the effect of gravity.
For simplicity, a simple geometry is considered in this study (tank of constant section A and depth z_L, fed at the rate Q_f at the level z_f).

The mathematical model is based on a mass balance PDE for the solid particles (in concentration C) settling down with a sedimentation velocity v_s :

$$\frac{\partial C}{\partial t}+\frac{\partial\left(C v_s + Cq\right)}{\partial z} = C_f q_f \delta\left(z - z_f\right)$$

Gravitational solid flux F_S — Hydraulic solid flux F_H

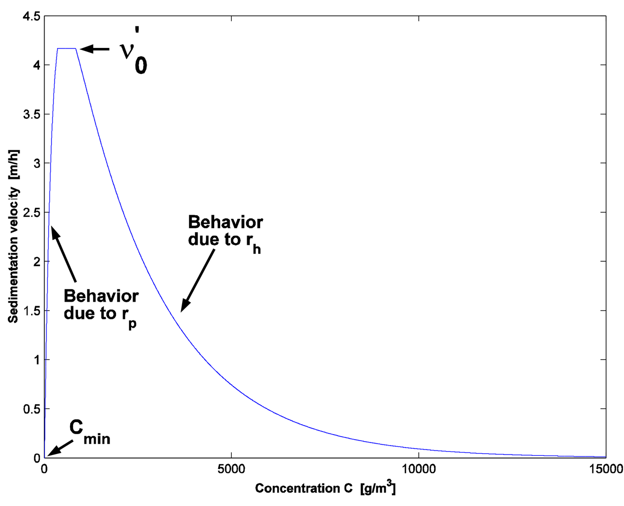
$$q = \begin{cases} -q_W & \text{for the upper part of the settler (above the feed level)} \\ q_S & \text{for the lower part of the settler (below the feed level)} \end{cases}$$

Law of sedimentation velocity

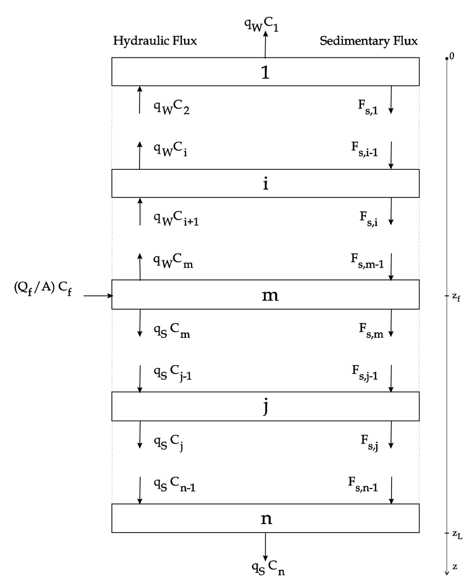
Takács's law possesses the desired features :

$$v_s = \max\left(0, \min\left(v_0^{'}, v_0\left(e^{\left[-r_h\left(C - C_{min}\right)\right]} - e^{\left[-r_p\left(C - C_{min}\right)\right]}\right)\right)\right)$$

The velocity increases because of the gravity acceleration exerted on the solid particles, reaches a maximum and then decreases because these particles are hindered by the other ones (formation of the sludge).



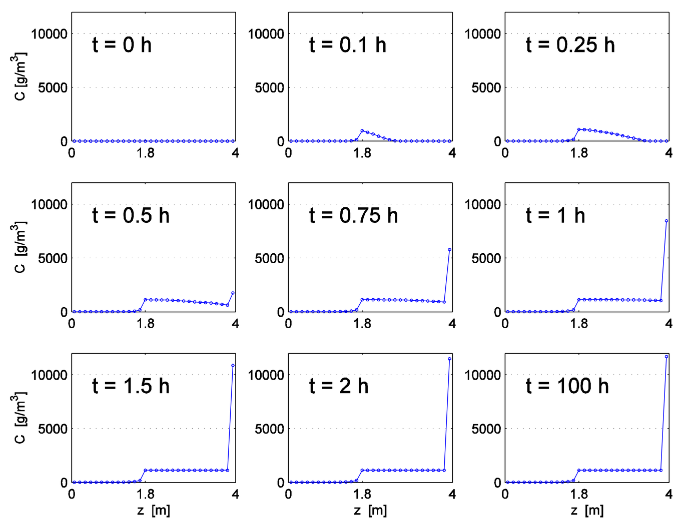
"Mixed-tank-in-series" formulation



The traditional "mixed-tank-in-series" formulation amounts to a discretization of the mass balance PDE with a first-order finite volume method.
The device is spatially cut in n layers of equal thickness, considered like little tanks on which a mass balance is calculated.

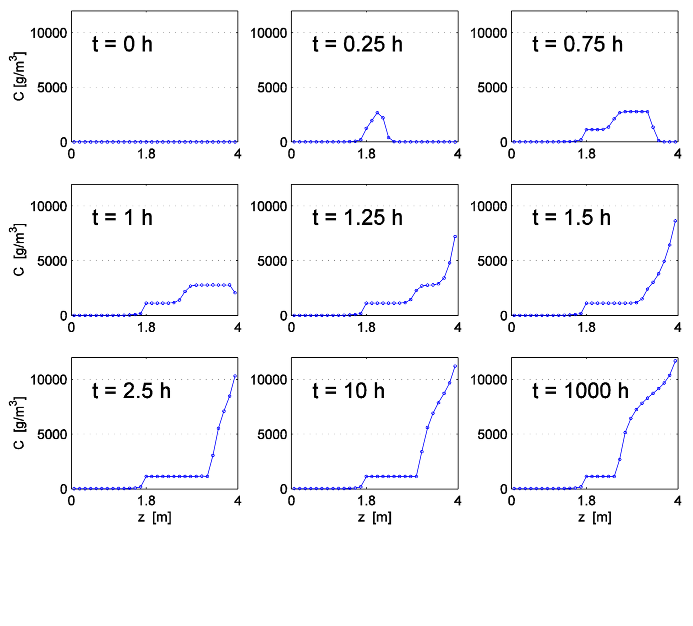
Model of Kynch

The first PDE model was proposed by G. Kynch in 1952, as an extension of Stoke's law.
After some time, the predicted spatial concentration profile become uniform in the lower part of the settler because the gravitational flux F_s is constant along this part.
However, a sharp spatial transition appears at the outlet boundary, which is not realistic.



Model of Takács

Takács introduced a condition on the gravitational flux F_s, as an ad-hoc procedure applicable to the "mixed-tank-in-series" formulation, based on the comparison between F_s values for successive layers.



In the upper zone :

$$F_{s,i} = \min\left(v_s(C_i).C_i, v_s(C_{i+1}).C_{i+1}\right) \quad \text{if} \quad C_{i+1} > C_t$$

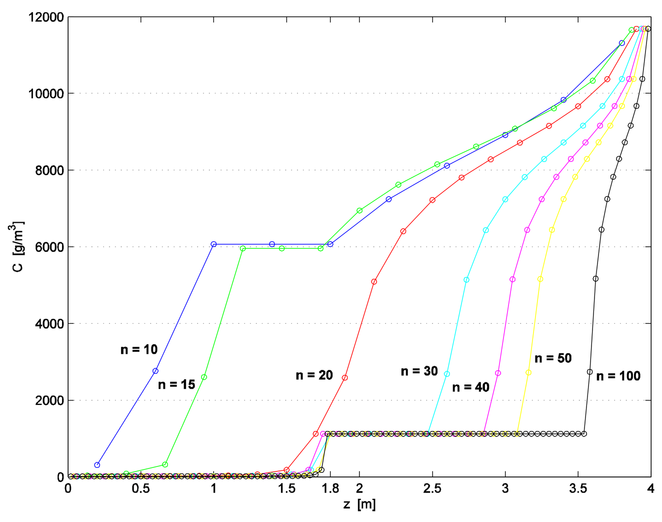
$$F_{s,i} = v_s(C_i).C_i \quad \text{if} \quad C_{i+1} \le C_t$$

where C_t is a threshold concentration

In the lower zone :

$$F_{s,i} = \min\left(v_s(C_i).C_i, v_s(C_{i+1}).C_{i+1}\right)$$

The simulation results are more realistic (accumulation of sludge), but there is an interplay between model formulation (conditions on F_s) and spatial discretization (number of layers), whereas both should be kept separate.



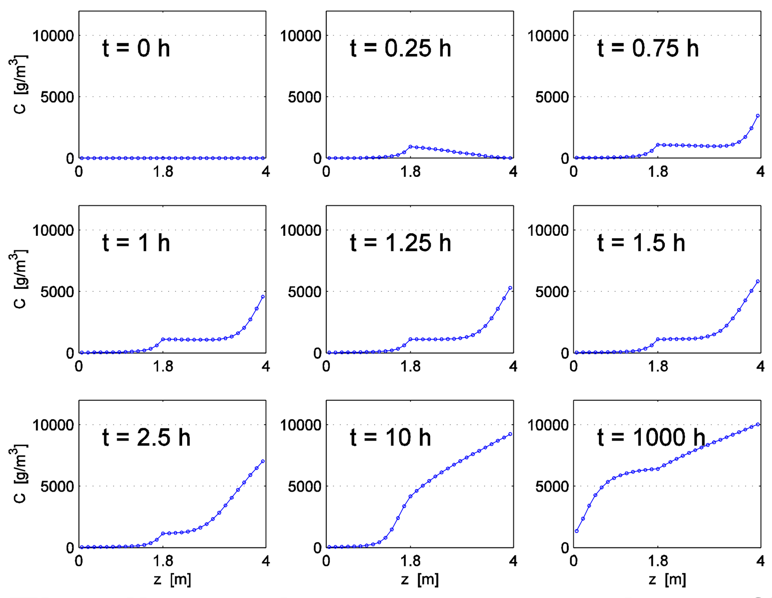
The predicted concentration profile changes when the number of layers n is increased, and there is no convergence, at least to the apparently defined physical profile, but to the concentration profile predicted by Kynch's model !

Model of Hamilton

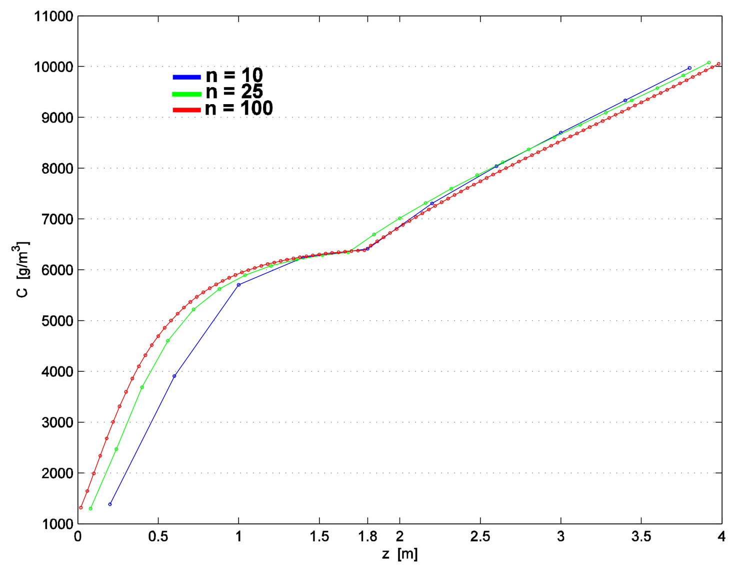
Hamilton introduced a term accounting for material dispersion in Kynch's model:

$$\frac{\partial C}{\partial t}+\frac{\partial\left(C v_s + Cq\right)}{\partial z} - D\frac{\partial^2 C}{\partial z^2} = C_f q_f \delta\left(z - z_f\right)$$

Diffusion coefficient



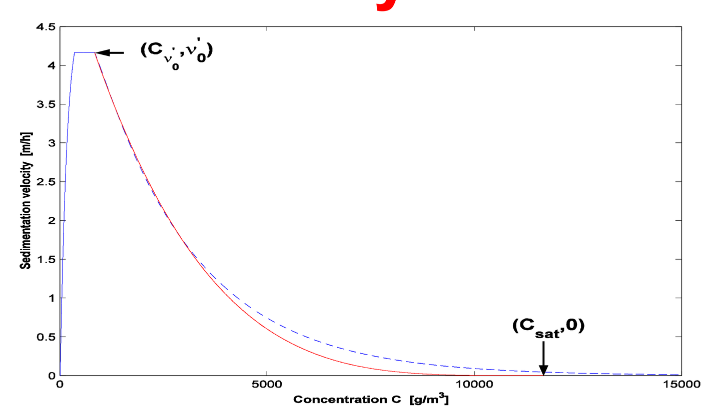
The dispersion term smoothes off the travelling waves and gives realistic concentration profiles.



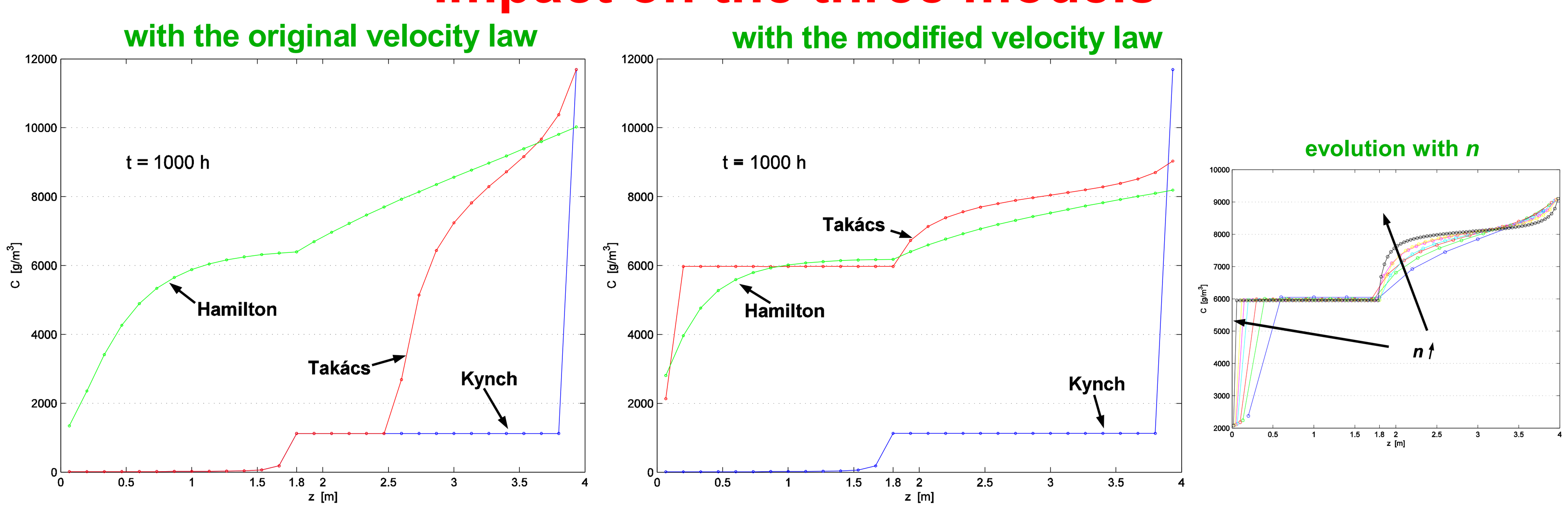
Numerical results converge for increasing numbers of layers.

Modified law of sedimentation velocity

The original law implies that for v_s=0, C is infinite. Physically, the sedimentation velocity should vanish at a finite concentration C_{sat}, representing the concentration at which the sludge forms a solid aggregate.
The exponential law is replaced by a parabola.



Impact on the three models



The biggest change is for Takács' model, which now gives results comparable to those obtained with Hamilton's model. In addition, a normal convergence is also observed when n is increased. A sharp, unrealistic, transition is observed at the top of the settler though, which is the consequence of the discretization method and the condition on the solid flux.

Method of Lines Solution

Hamilton's PDE model, with a modified settling velocity, is solved using the Method of Lines, i.e. using higher-order finite difference methods (implemented as differentiation matrices in a MATLAB toolbox MATMOL).

$$\begin{cases} \frac{\partial C_1}{\partial t}+\frac{\partial\left(C_1 v_{s,1} + C_1(-q_W)\right)}{\partial z_1} - D\frac{\partial^2 C_1}{\partial z_1^2} = 0 & \text{for the upper part} \\ \frac{\partial C_2}{\partial t}+\frac{\partial\left(C_2 v_{s,2} + C_2 q_S\right)}{\partial z_2} - D\frac{\partial^2 C_2}{\partial z_2^2} = 0 & \text{for the lower part} \end{cases}$$

Boundary and continuity equations are imposed at the top, feed and bottom levels of the tank:

$$\begin{cases} \text{for } z = 0: \frac{\partial C_1}{\partial z_1} = 0 \\ \text{for } z = z_f: q_f C_f + \left(v_{s,1} - q_W\right)C_1 - \left(v_{s,2} + q_S\right)C_2 = 0 \\ \text{for } z = z_L: v_{s,2} = 0 \end{cases}$$

n_1 = 76 and n_2 = 76