Metaheuristics for multiobjective combinatorial optimization problems

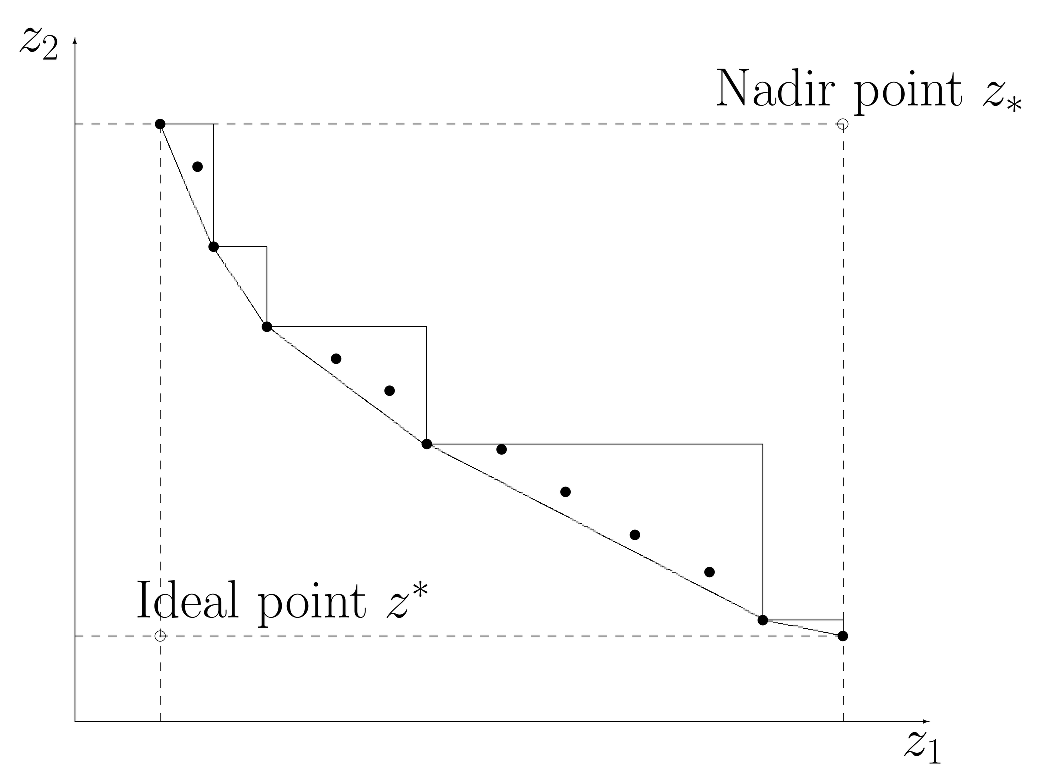
T. Lust, J. Teghem
Faculté Polytechnique de Mons, Belgium

Aim

A multiobjective combinatorial optimization problem with K objectives, n variables and q constraints is defined in the following way:

$$\begin{bmatrix} \ll \min_{X \in D} \gg & z_k(X) = c^k X & k = 1, \ldots, K \\ \text{where} & D = \{X : AX \leq b, X \in \{0,1\}^n\} \end{bmatrix}$$

Due to the contradictory features of the objectives, it does not exist a solution simultaneously minimizing each objective, but a set of solutions called *efficient solutions*. A solution $X^* \in D$ is efficient if there are no other solutions $X \in D$ such as: $z_k(X) \leq z_k(X^*), k = 1, \ldots, K$ with at least a strict inequality.



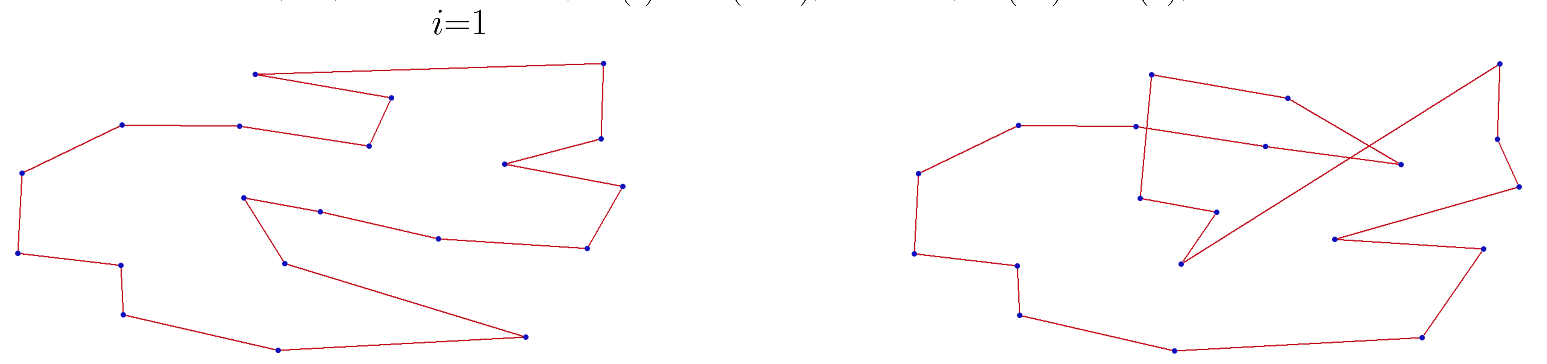
Presentation of MEMOX

The resolution scheme of multiobjective optimization problems, called `MEMOX`, is based on a memetic algorithm. A memetic algorithm is a genetic algorithm where the mutation operator is replaced by a local search method applied to every new offspring generated.
The particularity of the method, compared to the other existing multiobjective memetic algorithms, is that the first parent selected, called X_1, is one of the solutions of the population having a minimal density, defined by the number of solutions being in the same hypervolume as the solution. We use a dynamic hypergrid, which evolves according to the average density of the solutions, to realize the division of the objective space in hypervolumes. The second one, called X_2, is one of the solutions of the potentially efficient solutions set among the closest solutions to the first solution X_1 according to the euclidean distance in the objective space.

Resolution of the biobjective TSP

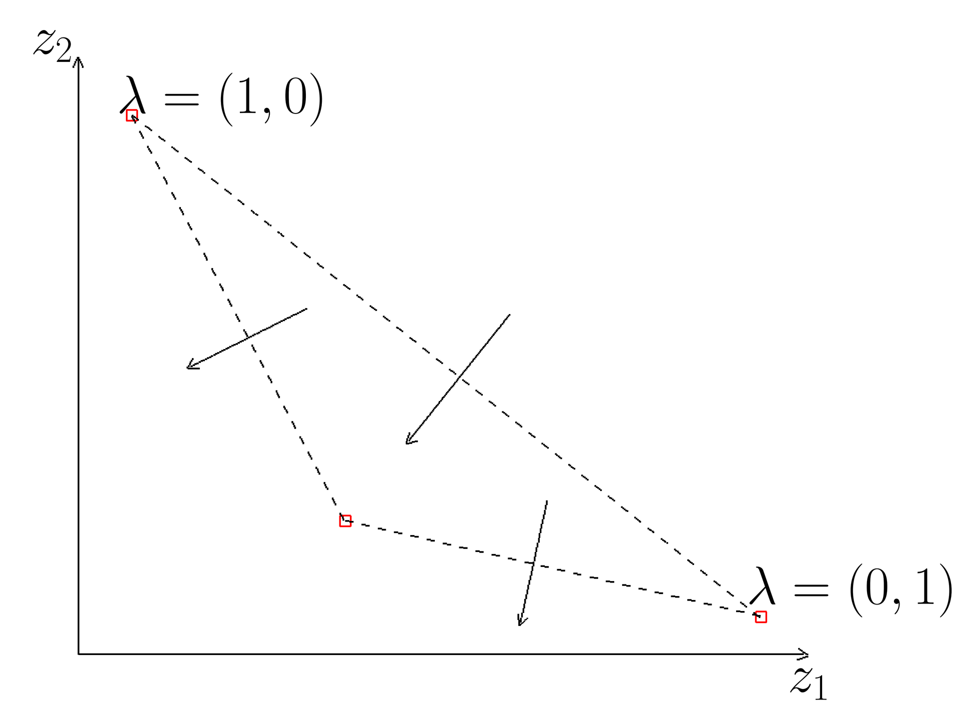
- The biobjective TSP

$$\ll \min \gg z_k(X) = \sum_{i=1}^{N-1} c_k(v_{\pi(i)}, v_{\pi(i+1)}) + c_k(v_{\pi(N)}, v_{\pi(1)}) \quad k = 1, \ldots, K$$

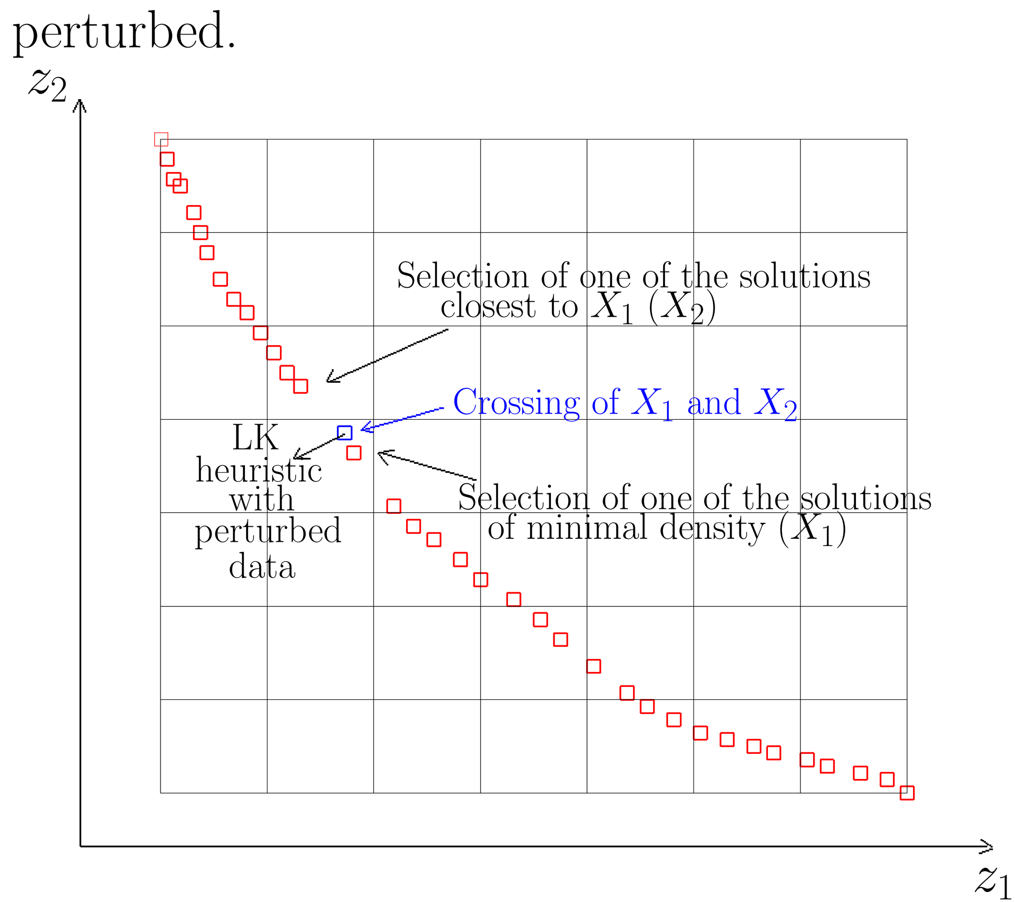


We have adapted the resolution scheme `MEMOX` to the resolution of the biobjective TSP. The Lin-Kernighan (`LK`) heuristic is used as local search, and we call the method resulting from the integration of the `LK` heuristic in the `MEMOX` scheme, `MEMO-LK`. Three phases are run independently:

- Phase 1: Generation of the initial population, essentially composed of supported efficient solutions, with the method of Aneja and Nair. This method consists in generating all the weight sets which make it possible to obtain a minimal complete set of extremal supported efficient solutions of a biobjective problem. The resolution of the uni-objective problems resulting of the linear aggregation is realized with the Lin-Kernighan heuristic.



- Phase 2: Main iterations of `MEMO-LK`: application of the `LK` heuristic starting from offsprings with perturbed data. As the `LK` heuristic is very robust (very little influenced by the starting solution), few new solutions will be found by the local search based on a linear aggregation, since a search for supported efficient solutions has already been launched during the initial population generation. We thus use the data perturbation (DP) technique: instead of modifying the starting solution (as carried out, for example, in the Iterated Local Search method), DP suggests to modify input data. In this way, by application of the `LK` heuristic starting from offsprings with perturbed data, new solutions, essentially potentially non-supported efficient, could be found since the data given by the linear aggregation are perturbed.



- Phase 3: a Pareto local search method based on the 2-opt movement is applied on each potentially efficient solution found by the preceding phases: all non-dominated solutions obtained by a 2-opt movement are added to the potentially efficient solutions set.

Results

We have compared the performances of `MEMO-LK` with by using several indicators to measure the quality of the approximations and we obtained better results.