Localization of RFid tags by means of an antenna array: influence of the array shape

POLYTECH.MONS

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Goal of the project: Localization of RFid tags inside a warehouse

- System principle: Estimation of the direction of arrival and intensity of the tag response with an antenna array.
- Main difficulties to overcome:
  - Multiple propagation path
  - Low signal to noise ratio of the received signal ($SNR \approx 0$ dB)
  - Limited size of the array for cost reason

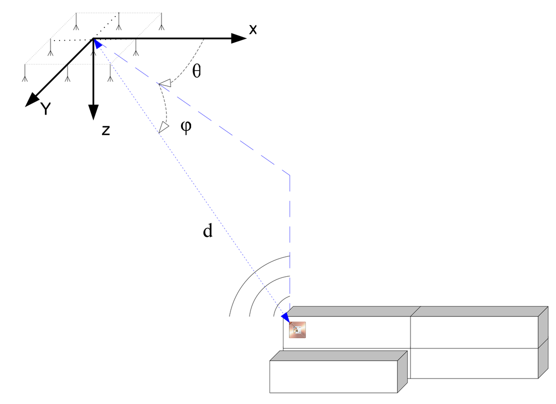

Figure 1 : RFTag – Localisation of RFid tags in warehouse

Direction of arrival estimation by a maximum likelihood based algorithm

1. **Propagation model**.

The presence of multipath components is simulated by delayed version of the signal, coming from different directions. Supposing that the tag response is a narrowband signal, the propagation model is given by:

$$x_k(t)=\frac{1}{d_0}g_k(\theta_0,\varphi_0)\cdot e^{j\frac{\omega_c}{c}(x_k\cos\theta_0\cos\varphi_0+x_k\sin\theta_0\cos\varphi_0)}s_0(t)+\sum_{i=1}^{N_R}\frac{1}{d_i}\alpha_i g_k(\theta_i,\varphi_i)\cdot e^{j\frac{\omega_c}{c}(x_k\cos\theta_i\cos\varphi_i+x_k\sin\theta_i\cos\varphi_i)}s_i(t)+n_k(t)$$

This model can be rewritten as : $\boldsymbol{x}(t)=\boldsymbol{A}(\boldsymbol{\theta},\boldsymbol{\varphi})\boldsymbol{s}(t)+\boldsymbol{n}(t)$

2. **Principle of the maximum likelihood estimation scheme**

→ To seek the parameters $(\boldsymbol{\theta},\boldsymbol{\varphi})$ maximizing the likelihood of the measured data according to the propagation model used:

$$p\left(\boldsymbol{x}(t)/\boldsymbol{\theta},\boldsymbol{\varphi},\sigma^2,s(t)\right)=\frac{1}{\pi^L}\cdot\frac{1}{(\sigma^2)^L}\cdot\exp\left(\left\|\boldsymbol{x}(t)-\boldsymbol{A}(\boldsymbol{\theta},\boldsymbol{\varphi})\boldsymbol{s}(t)\right\|^2/\sigma^2\right)$$

Under the hypothesis of independent snapshots, the function the estimation of the azimuth $\boldsymbol{\theta}$ and elevation $\boldsymbol{\varphi}$ angle are obtained by minimizing the function:

$$\min_{\boldsymbol{\theta},\boldsymbol{\varphi},\sigma^2,s(t)} l_{DML}\left(\boldsymbol{\theta},\boldsymbol{\varphi},\sigma^2,s(t)\right)=\min_{\boldsymbol{\theta},\boldsymbol{\varphi},\sigma^2,s(t)}\left\{L\log\sigma^2+\frac{1}{\sigma^2\cdot N}\sum_{i=1}^{N}\left\|\boldsymbol{x}(t)-\boldsymbol{A}(\boldsymbol{\theta},\boldsymbol{\varphi})\boldsymbol{s}(t)\right\|^2\right\}$$

Optimization of the antenna array shape

1. Antenna array = spatial sampling

➔ The shape of the array has a dramatic influence on the estimation performance.

➔ The larger array, the higher accuracy

BUT

➔ The cost of the array and the demodulation system is proportional to the number of antennas

➔ If all antenna are spaced with more than a half-wavelength, grating lobes appears and ambiguities occurs in the estimation process (Nyquist criteria for spatial sampling):

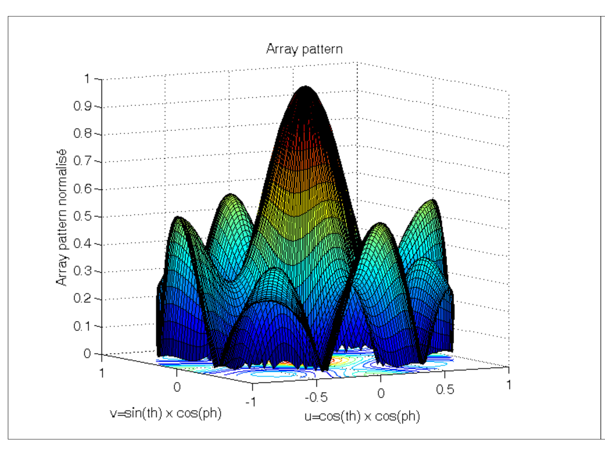

Figure 2 : Array pattern of an antenna array where all antennas are spaced by a half wavelength

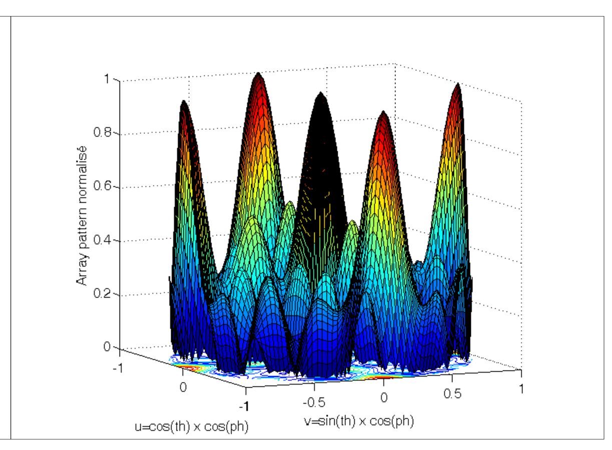

Figure 3 : Array pattern of an antenna array where all antennas are spaced by a wavelength

Under these constraints, numerous array shapes possible: antennas can be regularly spaced by $\frac{\lambda}{2}$ or irregularly disposed:

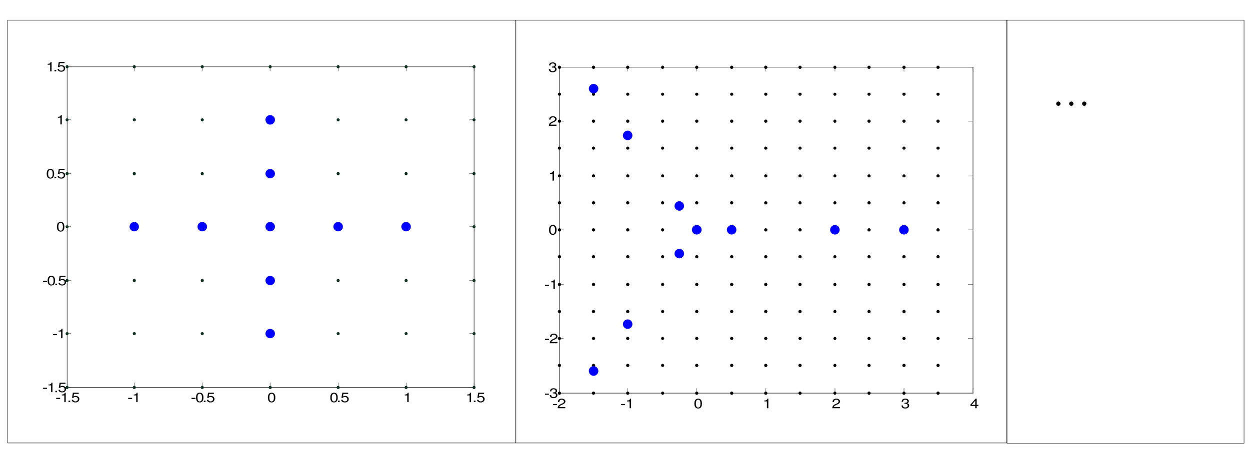

Figure 4 : Some examples of antenna array shape

→ Irregular arrays have a greater aperture but the array pattern presents higher sidelobes which can decrease the estimation accuracy at low SNR.

So, which one provides the best results for our problem?

2. General remarks on antenna disposition:

- Evolution of the estimation error with signal to noise ratio:

The DML algorithm for DOA estimation is an efficient estimation algorithm. For a number of snapshots sufficiently high, the estimation error attains the Cramer Lower Bound which is a theoretical lower bound on the variance estimation of any estimator.

However, below a certain signal to noise ratio, the error on the direction of arrival rises sharply: secondary lobes of the array pattern generates incorrect global maximum in the likelihood function. As a result, the antenna array system cannot be used below this SNR threshold.

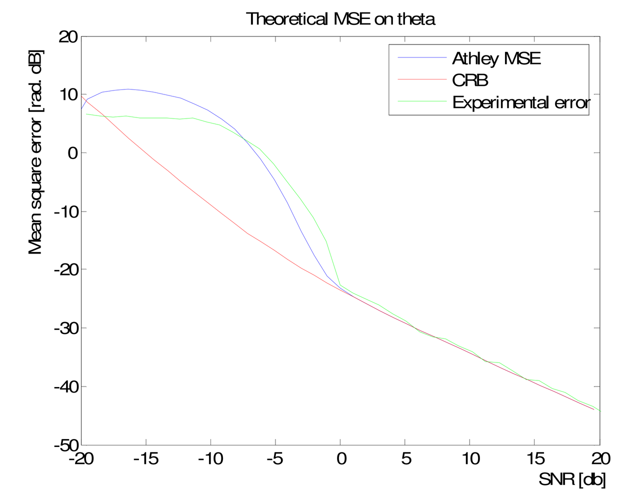

Figure 5 : Evolution of the theoretical and experimental mean square error and Cramer Rao Bound with the signal to noise ratio for the direction of arrival of one signal and 16 snapshots

- For a given array configuration, the estimation error varies with the direction of arrival of the incident signal. For example, a square array with 9 antennas regularly spaced exhibits performance degradation for very oblique signals. Less regular structures (like minimum redundancy array) or "Y" shape arrays can prevent such problem

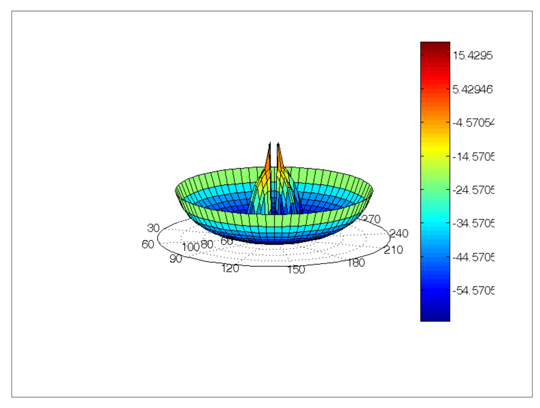

Figure 6 : Evolution of the MSE error on θ for 9 antennas square array

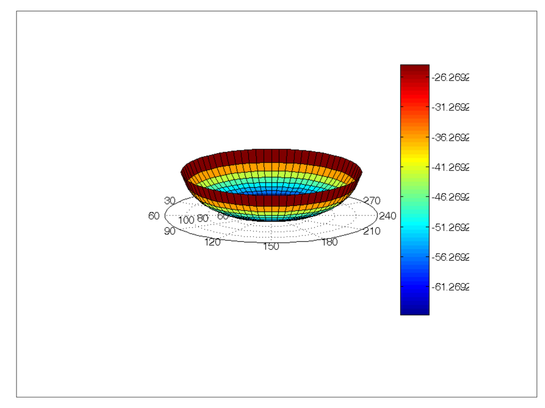

Figure 7 : Evolution of the MSE error θ on for 7 antennas "Y" array

3. Main characteristics used to compare array's

Two characteristics were used to compare array properties:

- Slope of the Cramer Rao Bound (representative of the estimation error at high SNR) (Figure 8)
- SNR threshold when using the DML algorithm (Figure 9)

As these characteristics vary with the direction of arrival, mean value on θ and φ were calculated and compared for different shape of array.

- **Calculation of the Cramer Rao Bound**

The Cramer Rao Bound on angle accuracy can be calculated from [2]:

$$E\left[\left(\hat{\theta}-\theta\right)\left(\hat{\theta}-\theta\right)^T\right]=\boldsymbol{R}_\varepsilon\geq\left[\frac{2N\cdot L\cdot SNR^2}{(1+L\cdot SNR)}\right]^{-1}\cdot\left[\left[\frac{\partial\boldsymbol{a}^H}{\partial\theta}\frac{\partial\boldsymbol{a}}{\partial\theta}-\frac{1}{L}\frac{\partial\boldsymbol{a}^H}{\partial\theta}\boldsymbol{a}\boldsymbol{a}^H\frac{\partial\boldsymbol{a}}{\partial\theta}\right]\right]^{-1}$$

As the second factor of the right expression regroups all dependencies from antennas position, it is used as performance indicator for a particular array.

- **Estimation of the SNR threshold**

A theoretical expression of the mean square error for likelihood estimation [1] is used to estimate the SNR threshold. In this expression, contributions from outliers and from local error are separately evaluated:

$$E\left[\left(\hat{\theta}-\theta\right)^2\right]=P[no\ outlier]E\left[\left(\hat{\theta}-\theta\right)^2 \mid no\ outlier\right]+P[outlier]E\left[\left(\hat{\theta}-\theta\right)^2 \mid outlier\right]$$

The SNR threshold is then defined as the signal to noise value at which the mean square error exceeds the Cramer Lower Bound by 10%.

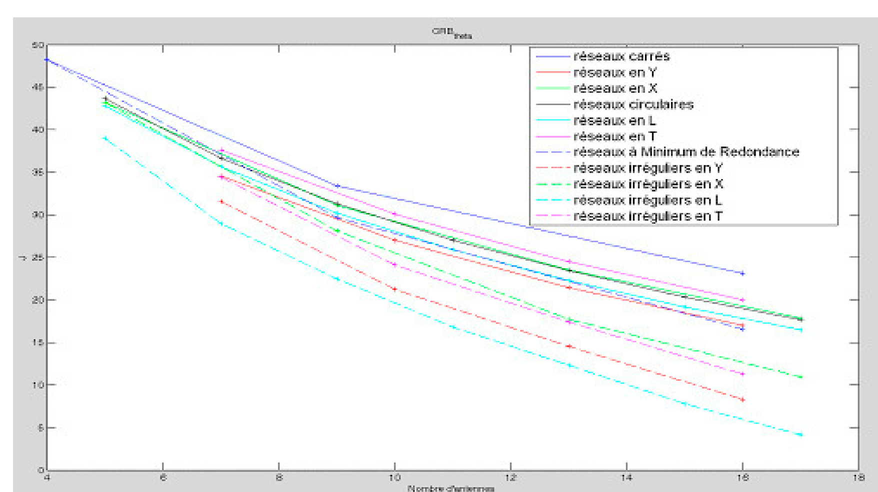

Figure 8 : Evolution of the slope of the Cramer Rao Bound on θ for different shape of array

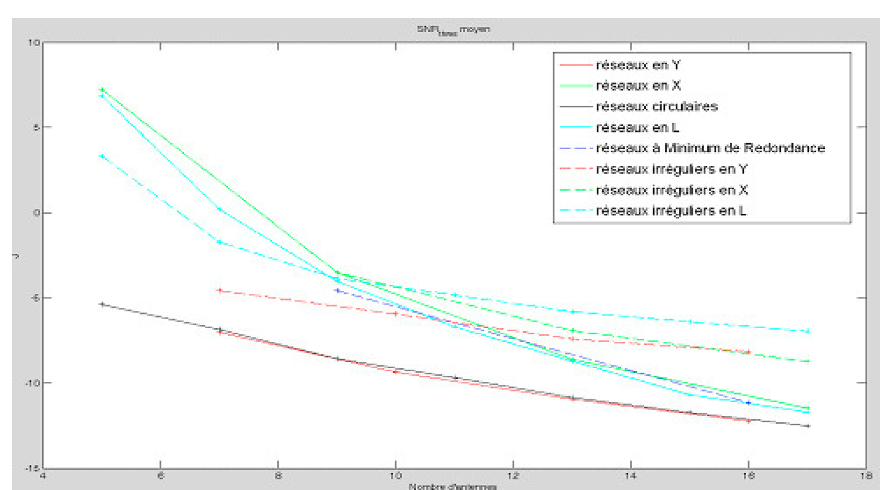

Figure 9 : Evolution the array SNR threshold on θ with the number of antenna for different shape of the array

Conclusion:

- For a given number of antennas, irregular arrays provide better accuracy at high signal to noise ratio.
- For a given number of antennas, the SNR threshold is generally lower for regular arrays than for irregular arrays.
- Some regular arrays like square and "L" shape arrays exhibit performance deterioration for very oblique signals. Simulations have shown that the use of "Y" arrays limits this problem.

As a result, a four antennas "Y" shape array has been selected for a first prototype.

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